NAME ______________________________ DATE ______________

5–12 Solving Equations by Factoring

Objective: To use factoring in solving polynomial equations.

Vocabulary

Zero-product property A product of factors is zero if and only if one or more of the factors is zero.

Polynomial equation An equation whose sides are both polynomials.

Linear equation A polynomial equation whose term of highest degree has degree 1. For example, $x - 2 = 0$ and $5x - 4 = 6$.

Quadratic equation A polynomial equation whose term of highest degree has degree 2. For example, $x^2 - x - 6 = 0$, $x^2 = 9x$, and $10x - 9 = x^2$.

Cubic equation A polynomial equation whose term of highest degree has degree 3. For example, $x^3 - 2x^2 + x - 1 = 0$.

Standard form of a polynomial equation A form of an equation in which one side is a simplified polynomial arranged in order of decreasing degree of the variable and the other side is zero.

Double or multiple root A factor that occurs twice in the factored form of an equation. For example, 5 is a double root of $x(x - 5)(x - 5) = 0$.

Text

Example 1 Solve $(x - 1)(x + 3) = 0$.

Solution Since the product of factors is 0, one of the factors on the left side must equal 0.

$$x - 1 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 1 \qquad\qquad x = -3$$

The solution set is $\{1, -3\}$. Just by looking at the original equation, you can see that when $x = 1$ or $x = -3$, the product will be 0.

Example 2 Solve $3n(n - 2)(n - 5) = 0$.

Solution

$$3n = 0 \quad \text{or} \quad n - 2 = 0 \quad \text{or} \quad n - 5 = 0$$
$$n = 0 \qquad\qquad n = 2 \qquad\qquad n = 5$$

The solution set is $\{0, 2, 5\}$.

CAUTION Never transform an equation by dividing by an expression containing a variable. Notice that in Example 2, the solution 0 would have been lost if both sides of the equation had been divided by $3n$.

Solve.

1. $(y + 4)(y - 5) = 0$

2. $0 = (n + 1)(n + 8)$

3. $10n(n - 2) = 0$

4. $2x(x - 10) = 0$

5. $(p - 1)(p - 7) = 0$

6. $0 = 2n(n - 1)(n - 3)$

7. $x(2x - 1)(2x + 1) = 0$

8. $0 = n(n - 6)$

9. $0 = 3x(4x - 1)(x - 2)$

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5–12 Solving Equations by Factoring *(continued)*

Example 3 Solve the quadratic equation $2x^2 - x = 3$.

Solution

1. Transform the equation into standard form. $2x^2 - x - 3 = 0$
2. Factor the left side. $(2x - 3)(x + 1) = 0$
3. Set each factor equal to 0 and solve.

$$2x - 3 = 0 \quad \text{or} \quad x + 1 = 0$$
$$2x = 3 \qquad x = -1$$
$$x = \frac{3}{2}$$

4. Check the solutions in the original equation.

$$2\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right) \stackrel{?}{=} 3$$
$$2\left(\frac{9}{4}\right) - \frac{3}{2} \stackrel{?}{=} 3$$
$$\frac{9}{2} - \frac{3}{2} \stackrel{?}{=} 3$$
$$\frac{9}{2} - \frac{3}{2} = \frac{6}{2} = 3 \ \checkmark$$

$$2(-1)^2 - (-1) \stackrel{?}{=} 3$$
$$2(1) + 1 \stackrel{?}{=} 3$$
$$3 = 3 \ \checkmark$$

The solution set is $\left\{-1, \frac{3}{2}\right\}$.

Solve.

10. $x^2 - x - 12 = 0$
11. $x^2 - 12x + 27 = 0$
12. $0 = x^2 - 4x - 32$
13. $0 = m^2 + 3m - 54$
14. $x^2 - 4y + 3 = 0$
15. $x^2 - 10x - 24 = 0$
16. $0 = n^2 - n$
17. $y^2 = 12y$
18. $6k^2 = 2k$
19. $x^2 + 16 = 8x$
20. $a^2 = 10 - 3a$
21. $3x^2 - x = 2$
22. $0 = x^2 + 12x + 35$
23. $y^2 + 5y = 14$
24. $x^2 = 5x + 36$
25. $4m^2 - 25 = 0$
26. $r^2 + 8 = 9r$
27. $6n^2 - n = 2$
28. $3x^2 + 1 = 4x$
29. $3a^2 = 6a$
30. $3p^2 - 14p = 80$
31. $2x^2 = 10 + x$
32. $3p^2 + 17p = -10$
33. $3x^2 + 1 = 4x$

Mixed Review Exercises

Evaluate if $x = 3$ and $y = 6$.

1. $(x - y)^3$
2. $x^3 \cdot x^2$
3. $4x^3$
4. $(4x)^3$
5. $3x + y^2$
6. $3x^2 + y$
7. $3(x + y)^2$
8. $(yx)^2$
9. y^2x^2

Simplify.

10. $(5x^2y^2)(-3xy^4)$
11. $(8a)^3$
12. $-3(x + 4)$